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A Study on Aperture Admittance of Rectangular Waveguide Terminated in Infinite Lossy Medium of Different Complex Permittivity Values

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***Abstract*- In this paper, variations of aperture admittance of a rectangular waveguide terminated in an infinite lossy medium are studied at 2.45 GHz through simulation using finite element method (FEM) based Ansoft's HFSS software and the simulation results are compared with theoretical results using plane wave spectral technique. The theoretical equation for aperture admittance was solved using MATLAB software. It is found that aperture admittance for this type of termination shows inductive behavior. Also, the simulated variations of aperture admittance with medium complex permittivity are nearly in agreement with the theoretical results. For the analysis and simulation purposes standard rectangular waveguide (WR-340) propagating only TE_{10} mode is used.**

***Index Terms*- Aperture admittance, dielectric permittivity, infinite lossy medium, rectangular waveguide.**

I. INTRODUCTION

With increasing use of waveguide based aperture antennas in radar, diathermy and hyperthermia, further studies on these types of antennas and accurate data on the dielectric properties of the materials are needed. The aperture antenna terminated in different types of dielectrics has been studied by many researchers using different approaches such as the variational, correlation matrix and integral solution methods [1-4]. MacPhie and Zaghloul proposed correlation matrix method based on the principle of energy conservation [3], while Gardiol and his group obtained the aperture admittance using an integral equation method based on the magnetic current concept [4]. Henri *et al.* [5] used a method based on the transverse field calculations through the use of operator called transverse operator. This method was introduced for the first time by Marcuvitz group [6]. In all the foregoing cases [1-6] the basic radiating structure is the open end of a waveguide, generally surrounded by an infinite metallic flange.

Stuchly *et al.* [7] proposed a new model for the aperture admittance of an open ended waveguide structure radiating into a homogeneous, lossy dielectric. Jan *et al.* [8] found that a rectangular aperture not more than one wavelength long in a conducting plane can be characterized by two complex numbers, called the aperture admittances for the regions on each side of the aperture. In reference [9] a new technique based on moment method [10], [11], named multiple cavity modelling technique, has been described for analyzing a class of waveguide aperture antennas and devices. Recently, Kim *et al.* [12] proposed a new formula for the calculation of reflection coefficient at the open end of a rectangular waveguide radiating into air.

The general problem of determining the admittance of a waveguide terminated in a lossy medium as a function of complex propagation constant 'k' in the lossy medium has been theoretically solved previously by Compton [13] for three common aperture sizes. Compton used

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plane wave spectral technique to derive the expression for aperture admittance and computed the values of aperture admittance with the help of analog computer.

The purpose of the present paper is to study through simulation the variation of aperture admittance of WR-340 standard rectangular waveguide terminated in an infinite lossy medium for different values of medium complex permittivity at 2.45 GHz and compare the simulation results with the theoretical results obtained using plane wave spectral technique [14]. The theoretical values of aperture admittance are obtained by solving the parameter equation through MATLAB software. The simulation results for aperture admittance for medium complex permittivity values of 13.85−j8 and 31.2−j8 are also compared with those obtained by Compton [13]. The simulation study has been carried out using FEM based Ansoft's HFSS software [15].

II. SIMULATION AND THEORETICAL BACKGROUND

A rectangular waveguide terminated in an infinite lossy medium is shown in Fig.1 along with the rectangular coordinate system. A conducting ground plane is also shown extending up to infinity in the aperture plane. The narrow and broad dimensions of the aperture of the rectangular waveguide are denoted as ‘a’ and ‘b’ respectively.

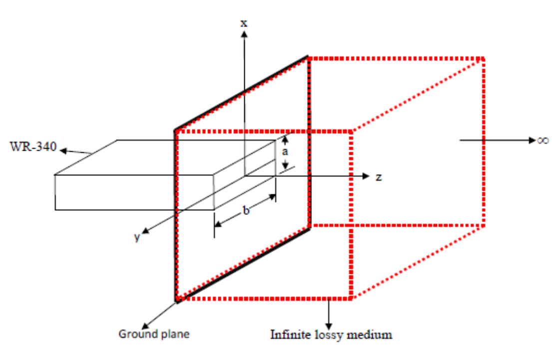

Fig.1. Rectangular aperture in a ground plane

A. Basics of Simulation

Various steps involved in the process of modeling and simulation using Ansoft's HFSS (High Frequency Structure Simulator) software are shown in Fig. 2 in the form of flowchart and are described briefly in the following. Ansoft's HFSS software uses the FEM (Finite Element Method) for electromagnetic analysis on arbitrary 3-D structures including antennas. FEM is a numerical technique for solving partial differential equations over complex domains. A continuous structure can be approximated as an assembly of finite elements. By formulating and combining the properties of these constituent elements, the solution for the entire structure can be obtained. In FEM, which is used in the software, the entire volume of the three dimensional structure is divided into a large number of tetrahedral elements. This software package can simulate a 2-D or 3-D object enclosed within any number of dielectrics.

B. Plane Wave Spectral Technique

The electric field in the aperture is assumed to correspond to the TE_{10} waveguide mode. The x-component of the electric field at the aperture of the rectangular waveguide is represented by

$$E_x(x,y,0) = \sqrt{\frac{2}{ab}}\cos\frac{\pi y}{b} \tag{1}$$

where, $\sqrt{\frac{2}{ab}}$ is amplitude coefficient.

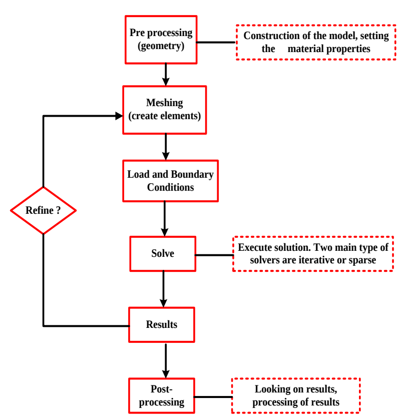

Fig.2. Flowchart for FEM numerical technique

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By using plane wave spectral technique [14], the x and y components of electric and magnetic fields in the infinite lossy medium respectively are given by [13]

$$E_x = \sqrt{\frac{2b}{a}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \cos\left(\frac{k_y b}{2}\right) \sin\left(\frac{k_x a}{2}\right) \left[\frac{1}{(\pi^2 - k_y^2 b^2)\pi k_x}\right] e^{-ik_y y} e^{-ik_x x} e^{-ik_z z} dx dy \quad (2)$$

$$H_y = \frac{1}{\omega\mu_0} \sqrt{\frac{2b}{a}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \cos\left(\frac{k_y b}{2}\right) \sin\left(\frac{k_x a}{2}\right) \left[\frac{k^2 - k_y^2}{(\pi^2 - k_y^2 b^2)\pi k_x k_z}\right] e^{-ik_y y} e^{-ik_x x} e^{-ik_z z} dx dy \quad (3)$$

where, k_x, k_y and k_z are the propagation constants along x, y and z directions respectively and k is the complex propagation constant in infinite lossy medium given by

$$k = \omega\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r} \quad (4)$$

where, ε_r and μ_r are the complex relative permittivity and permeability of the infinite lossy medium respectively. ω (= $2\pi f$) and f are the angular frequency and operating frequency of the wave respectively.

Through the use of Parseval's theorem [13], the aperture admittance can be expressed in terms of the spectral functions as follows:

$$Y_A = \frac{1}{2\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_x^* H_y dk_x dk_y \quad (5)$$

where, E_x^* is the complex conjugate of E_x

By substituting equations (2) and (3) and making use of Parseval's theorem and solving we get [13]

$$Y_A = \frac{1}{2\pi^2 k_z \omega\mu_0} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(k_x,k_y)^* (k^2 - k_y^2) f(k_x,k_y) dk_x dk_y \quad (6)$$

where, $f(k_x,k_y)$ is the Fourier transform of y component of electric vector potential given by [13]

$$f(k_x,k_y) = \frac{4\pi bi}{k_x k_z} \sqrt{\frac{2}{ab}} \sin\left(\frac{k_x a}{2}\right) \cos\left(\frac{k_y b}{2}\right) \left[\frac{1}{\pi^2 - b^2 k_y^2}\right] \quad (7)$$

The values of aperture admittance 'Y_A' for different complex permittivity values of the medium were evaluated using MATLAB software.

III. VALIDATION OF THE RESULTS OBTAINED THROUGH SIMULATION AND ANALYSIS

Theoretical and simulated results are verified at 2.45 GHz by estimating the values of aperture admittance of a rectangular waveguide carrying TE_{10} mode terminated in infinite lossy medium for different values of complex relative permittivity of the medium using plane wave spectral technique and comparing the theoretical results evaluated through MATLAB software with the simulated results using Ansoft's HFSS software and those obtained by Compton [13].

The theoretical values of admittance of the waveguide antenna of aperture dimensions, a = 6.1225 cm, and b = 3.06125 cm, terminated in an infinite lossy medium having two different complex permittivity values (= 13.85−j8 and 31.2−j18), obtained using plane wave spectral technique at 2.45 GHz are nearly in agreement with the simulated results obtained using Ansoft's HFSS software and those obtained by Compton [13] (Table 1).

Table 1: Aperture admittance of a rectangular waveguide carrying TE_{10}-mode terminated in infinite lossy medium at 2.45 GHz

Complex Relative permittivity of lossy slab	**Admittance (mho)**		
	Compton [13]	Plane wave spectral technique	HFSS
13.85−j8	0.0098−j0.0019	0.0099−j0.0019	0.0100−j0.0042
31.2−j18	0.0156−j0.0029	0.0152−j0.0031	0.0166−j0.0069

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IV. RESULTS AND DISCUSSION

For the simulation and analysis purposes standard waveguide WR-340, terminated in an infinite lossy medium is used in which an infinite conducting ground plane in the aperture plane is present. The dielectric property of the infinite lossy medium is represented by $\varepsilon_r = \varepsilon^{'} - j\varepsilon^{''}$. The HFSS model for estimation of aperture admittance of the waveguide terminated in infinite lossy medium is shown in Fig. 3.

Figs. 4 - 6 show the effect of varying $\varepsilon^{'}$ over the range 40−60 on the value of aperture admittance by taking $\varepsilon^{''}$ as a parameter. It can be seen from Figs. 4 - 8 that the results for aperture admittance obtained through simulation deviates somewhat from those obtained using plane wave spectral technique. This deviation in the results may be due to the assumption made in plane wave spectral technique that the ground plane at the aperture extends up to infinity. It is clear from Figs. 4 - 6 that the values of both aperture conductance and susceptance obtained through simulation increase at slow pace with $\varepsilon^{'}$ which seems logical due to more confinement of the wave within the medium and hence more energy storage capacity of the medium as $\varepsilon^{'}$ increases. The rate of increase of conductance with $\varepsilon^{'}$ is more than the rate of change of the susceptance.

Similarly, Figs. 7 and 8 show the variation of aperture admittance versus $\varepsilon^{''}$ over the range of 12 - 20 by taking $\varepsilon^{'}$ as a parameter. It is apparent from the curves of Figs. 7 and 8 that the conductance is quite sensitive to the imaginary part of the dielectric constant $\varepsilon^{''}$ of the medium, since conductivity of an isolated infinite medium is directly proportional to $\varepsilon^{''}$. The aperture susceptance is inductive and reduces with increase in $\varepsilon^{''}$ value of the medium. This shows reducing energy storage capacity of the medium and hence more energy dissipation in the medium with increase in the $\varepsilon^{''}$ value of the medium.

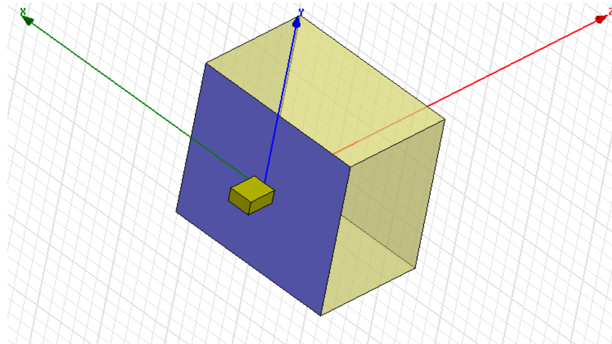

Fig.3. HFSS model to estimate the aperture admittance

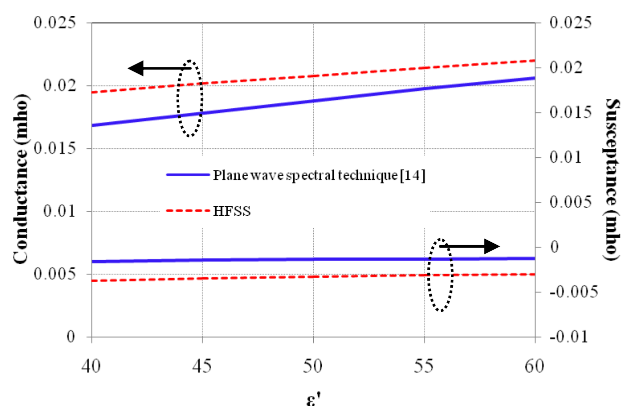

Fig.4. Effect of ε' on the aperture admittance for ε''=11

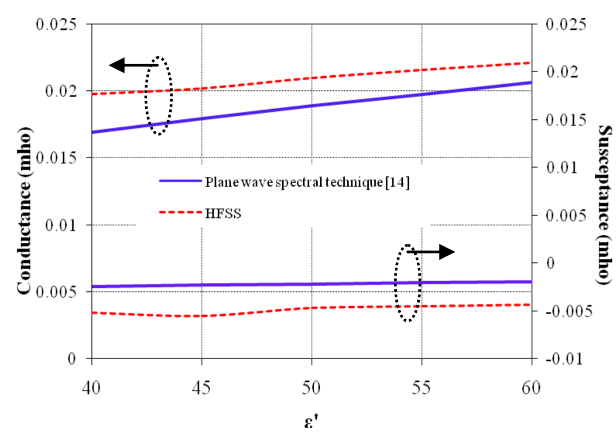

Fig.5. Effect of ε' on the aperture admittance for ε''=15

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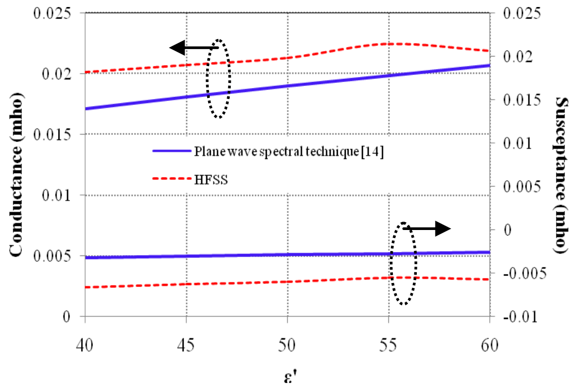

Fig.6. Effect of ε' on the aperture admittance for ε''=19

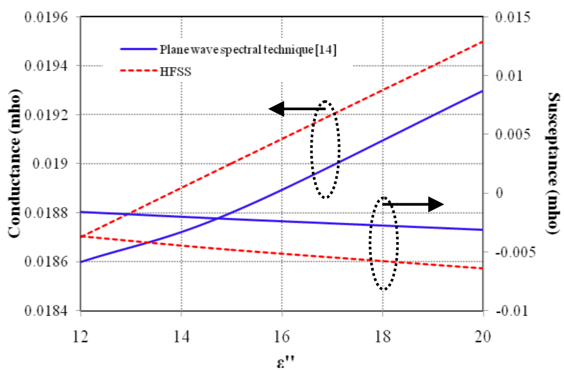

Fig.7. Effect of ε'' on the aperture admittance for ε'=47.5

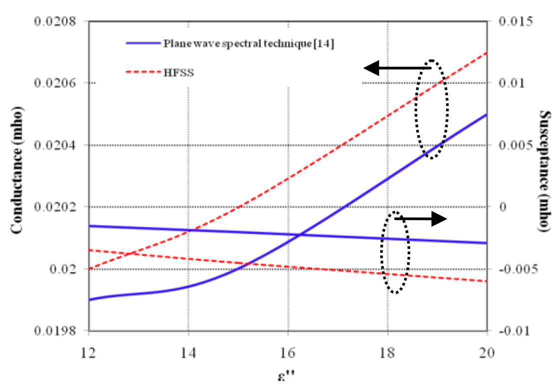

Fig.8. Effect of ε'' on the aperture admittance for ε'=55

V. CONCLUSION

The admittance of a rectangular aperture surrounded by infinite conducting ground plane and terminated in the infinite lossy medium has been determined at 2.45 GHz by using Ansoft's HFSS software for different complex permittivity values of the medium and the simulated results have been compared with the theoretical results obtained using plane wave spectral technique. Fair agreement in the results has been achieved. The simulated results have been validated with a few results obtained by Compton [13]. The results presented in this paper would help in quantifying the absorption behavior of microwave energy in the bio-medium/bio-layer.

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